and $\theta < 70°$.

Figure 29.5 shows the muon energy spectrum at sea level for two angles. At large angles low energy muons decay before reaching the surface and high energy pions decay before they interact, thus the average muon energy increases. An approximate extrapolation formula valid when muon decay is negligible ($E_\mu > 100/\cos\theta$ GeV) and the curvature of the Earth can be neglected ($\theta < 70°$) is

$$\begin{aligned}\frac{dN_\mu}{dE_\mu d\Omega} \approx & \frac{0.14\, E_\mu^{-2.7}}{\text{cm}^2 \text{ s sr GeV}} \\ & \times \left\{ \frac{1}{1 + \frac{1.1 E_\mu \cos\theta}{115\,\text{GeV}}} + \frac{0.054}{1 + \frac{1.1 E_\mu \cos\theta}{850\,\text{GeV}}} \right\}\end{aligned} \tag{29.4}$$

where the two terms give the contribution of pions and charged kaons. Eq. (29.4) neglects a small contribution from charm and heavier flavors which is negligible except at very high energy [64].

The muon charge ratio reflects the excess of $\pi^+$ over $\pi^-$ and $K^+$ over $K^-$ in the forward fragmentation region of proton initiated interactions together with the fact that there are more free and bound protons than free and bound neutrons in the primary spectrum. The increase with energy of $\mu^+/\mu^-$ shown in Fig. 29.6 reflects the increasing importance of kaons in the TeV range [65] and indicates a significant contribution of associated production by cosmic-ray protons ($p \to \Lambda + K^+$). The same process is even more important for atmospheric neutrinos at high energy.